0.0.1 Report of Project Part 7

Title: Pattern and 3D Shape Recognition

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Summary

In this project *pattern* and *shape recognition* have been investigated from *partial differential equations (PDEs)* and *inverse problems* points of view.

In the fundamental and inspiring book of Y. Meyer [Mey01] patterns are characterized as oscillating functions, which in turn are considered elements of dual Sobolev spaces. The concept of oscillating patterns can be used for pattern recognition and edge enhancing techniques with regularization methods exploiting the Bregman distance, a concept which has been established by Burger et al. Recently, we have extended this approach to texture enhancing.

Manay et al. have introduced the research area of signatures a few years ago. Since then *integral invariants* and according *signatures* have been identified to be useful for shape classification, which is an important research topic in computer vision, artificial intelligence and pattern recognition. Integral invariants and signatures are transformations of shapes. In general, the invariants are constructed in such a way that they are invariant under geometric transformations and allow for a compact representation of shapes. Our joint collaboration with H. Pottmann was to identify the *inverse problems* point of view of integral invariants, which is a core research area of the *Infmath Imaging* group.

Filtering of high-dimensional data, such as color data, is an active research area in PDEs. In the computer vision areas research is heavily driven by modeling of such differential equations for high-dimensional data. We emphasize that there are much less publications concerned with variational filtering methods. Our work is concerned with generalizing *morphological partial differential equations* for analysis of intensity data to high-dimensional data. The challenging task is to establish a solution concept with a rigorous mathematical analysis. The difficulty is that the concept of viscosity solutions, which applies to morphological differential equations, does not generalize to high-dimensional data.

Scientific Background / State of the Art

The modeling of integral invariants and signatures for shape analysis has not attracted attention in the inverse problems community so far. In the beginning our joint work with H. Pottmann was dedicated to identifying a novel research area in *inverse problems*. Thereby, we addressed the *inverse problem* point of view of integral invariants and signatures and highlighted some fundamental mathematical question. Extremely challenging and interesting mathematical problems could be raised. The inverse problems point of view added some novel perspectives to this research area, in particular novel designs of shape invariants.

Concerning the analysis and modeling of morphological partial differential equations and variational principles for data analysis on *manifolds* and in *high-dimensional spaces*, we have developed the concept of *non-convex* semigroups. Morphological partial differential equations for intensity data are commonly analyzed for viscosity solutions. For the morphological partial differential equations studied in image processing the concept of non-convex semigroups applies and gives generalized solutions. The advantage of our approach is that it also applies to high-dimensional data.

Results and Discussion

Integral Invariants

For *shape matching* and *classification*, an object, given by its shape, is compared with representative shapes of classes within a database. These problems are addressed in many areas of applied sciences such as *computer vision*, *artificial intelligence*, and *pattern recognition*.

The problems are tackled by a-priori assigning each object class within a database one or more typical representatives that capture the dominant features of the class. Then, the object under investigation is compared with the representative shapes using an appropriate notion of similarity. Common distance measures, such as the Hausdorff distance, are not appropriate, since they do not take into account the significance of dominant features. Additional shortcomings of global distance measures result from the fact that they are not invariant with respect to rigid motions.

We consider descriptors of shapes which emphasize on peaks, edges, or ridges. These features can be expressed by differentials of the shape boundary (and thus are invariant under rigid body transformations). Differentials have been used successfully for shape matching and classification, but are difficult to handle numerically, since they are unstable with respect to noise. Alternatively, *integral invariants* have been proposed by Manay et al. [MHYS04]. In comparison with differential invariants, a significant advantage of integral invariants is that by adopting the integration kernel they can be used selectively to capture dominating scales (features).

In this NRN the groups of Pottmann and Wallner have contributed to the geometrical point of view of integral invariants and signatures. Moreover, in geometry, integral invariants have proven to be practically relevant for object classification and shape matching.

However, from an inverse problem point of view, important questions have not been addressed so far. We have pointed out open questions and related this area to the problem of inverting spherical means.

The first question – from an inverse problems point of view probably the most important one – is the theoretical possibility to uniquely reconstruct a shape from its integral invariant. A positive answer implies that an integral invariant uniquely determines a shape, and thus can be considered as a token. This question is even more involved when only partial data of integral invariants of an object are available; such a handicap can be observed if the object is partially occluded during data acquisition or the data of integral invariants has been damaged. The second question concerns stability and the effect of noise on the shape matching and classification process. For this purpose appropriate distance measures of integral invariants have to be determined. Recently, we have developed novel variants of integral invariants for which partial answers to these questions could be derived. However, in the general framework these questions could not be solved.

Vector Valued Denoising

This work is concerned with *variational methods* and *partial differential equations* for denoising of *vector (tensor and matrix) valued data* $\vec{u}_0 : \Omega \subseteq \mathcal{R}^n \to \mathcal{R}^m$. Tensors and matrix valued data are considered as vectors, where matrix constraints – such as symmetry – are imposed as constraints on the function space setting.

The existing literature on diffusion filtering for denoising and enhancing of vector valued data is vast, in particular filtering of color images has attracted much attention. Much less publications are concerned with variational methods for filtering of high-dimensional data.

Following literature, we differ between three concepts based on evolutionary partial differential equations for filtering of tensor valued data $\vec{u}_0$.

The evolution equation for data filtering is defined by the steepest descent direction of a **convex** energy functional $J(\cdot)$ on a suitable function space. In general, denoting by ∂J the subdifferential (i.e., the descent direction) of J, these evolution equations are inclusion equations, and read as follows:

$$\begin{aligned} \frac{\partial \vec{u}}{\partial t}(\vec{x},t) &\in \partial J(\vec{u})(\vec{x},t)\,, \quad (\vec{x},t) \in \Omega \times (0,T)\,, \\ \vec{u}(\vec{x},0) &= \vec{u}_0(\vec{x})\,, \quad \vec{x} \in \Omega\,. \end{aligned} \tag{1}$$

This approach has been considered by Blomgren & Chan [BC96], with the total variation seminorm functional J. For intensity data u_0 (that is if $m = 1$), *mean curvature motion* (MCM) reads as follows

$$\begin{aligned} \frac{\partial u}{\partial t}(\vec{x},t) &= |\nabla u|\, \nabla \cdot \left(\frac{\nabla u}{|\nabla u|}\right)(\vec{x},t)\,, \quad (\vec{x},t) \in \Omega \times (0,T)\,, \\ u(\vec{x},0) &= u_0(\vec{x})\,, \quad \vec{x} \in \Omega\,. \end{aligned} \tag{2}$$

This can be interpreted as a diffusion process along the direction orthogonal to the gradient: Let ζ be a normalized vector normal to ∇u, then the MCM reads as follows

$$\begin{aligned} \frac{\partial u}{\partial t}(\vec{x},t) &= \partial_{\zeta\zeta} u(\vec{x},t)\,, \quad (\vec{x},t) \in \Omega \times (0,T)\,, \\ u(\vec{x},0) &= u_0(\vec{x})\,, \quad \vec{x} \in \Omega\,, \end{aligned} \tag{3}$$

where $\partial_{\zeta\zeta} u$ denotes the second derivative of u in direction ζ. This idea was extended for filtering of vector valued data $\vec{u}_0$ by defining ζ as an eigenvector corresponding to the smallest eigenvalue of $\nabla \vec{u}$ and defining

$$\begin{aligned} \frac{\partial \vec{u}}{\partial t}(\vec{x},t) &= \partial_{\zeta\zeta} \vec{u}(\vec{x},t)\,, \quad (\vec{x},t) \in \Omega \times (0,T)\,, \\ \vec{u}(\vec{x},0) &= \vec{u}_0(\vec{x})\,, \quad \vec{x} \in \Omega\,. \end{aligned} \tag{4}$$

Chambolle [Cha94] probably has been the first to publish this idea and many authors have followed and extended this idea (see for instance Sapiro & Ringach [GR96], Kimmel, Malladi & Sochen [KMS00], Weickert [Wei98], Whitacker & Gerig [WG94], Tang, Sapiro & Caselles [TSC00], Feddern, Weickert, Burgeth & Welk [FWBW06], to name but a few). Inspired by diffusion tensor medical imaging Burgeth [BBD+06] derived morphological evolution filtering methods for matrix valued data, which preserve matrix structures like symmetry and positive definiteness. Constrained preserving vector valued data filtering has also been considered by Tschumperle & Deriche [TD01, TD02] who considered diffusion filtering of vector valued data on the unit sphere. In the latter approach the designed filtering techniques are to preserve this feature by implementing two step algorithms with a descent algorithm and projection on the admissible data.

The goal of this work has been to derive evolutionary partial differential equations for filtering of vector valued data as asymptotical limits of non-convex variational principles. In principle we proceed as in Blomgren & Chan [BC96] and derive an evolution equation from an energy minimization principle. However, the difference is that our energy functional is non-convex, and therefore semigroup theory (see [CL71]) as a mathematical foundation for the diffusion filtering method does not apply. We propose a concept of *non-convex semigroups* which generalizes the notion of convex semigroup theory, and discuss some of the paradox which are inherent in this solution concept. This concept is also considered in Subproject 03, but there we concentrate on level set evolutions and intensity data. Although in many perspectives the mathematical analysis is similar, there exist fundamental differences which have to do with generalizing the concept of convexification to high-dimensional data. The derived evolution equations for vector valued data can be considered generalizations of the MCM equation for filtering of intensity data.

National and International Cooperation

Cooperations have been performed with H. Pottmann on integral invariants and a new area for inverse problems has been discovered. Together with P. Elbau (Radon Linz) and G. Dziuk (Freiburg) we collaborated on high-dimensional data filtering.

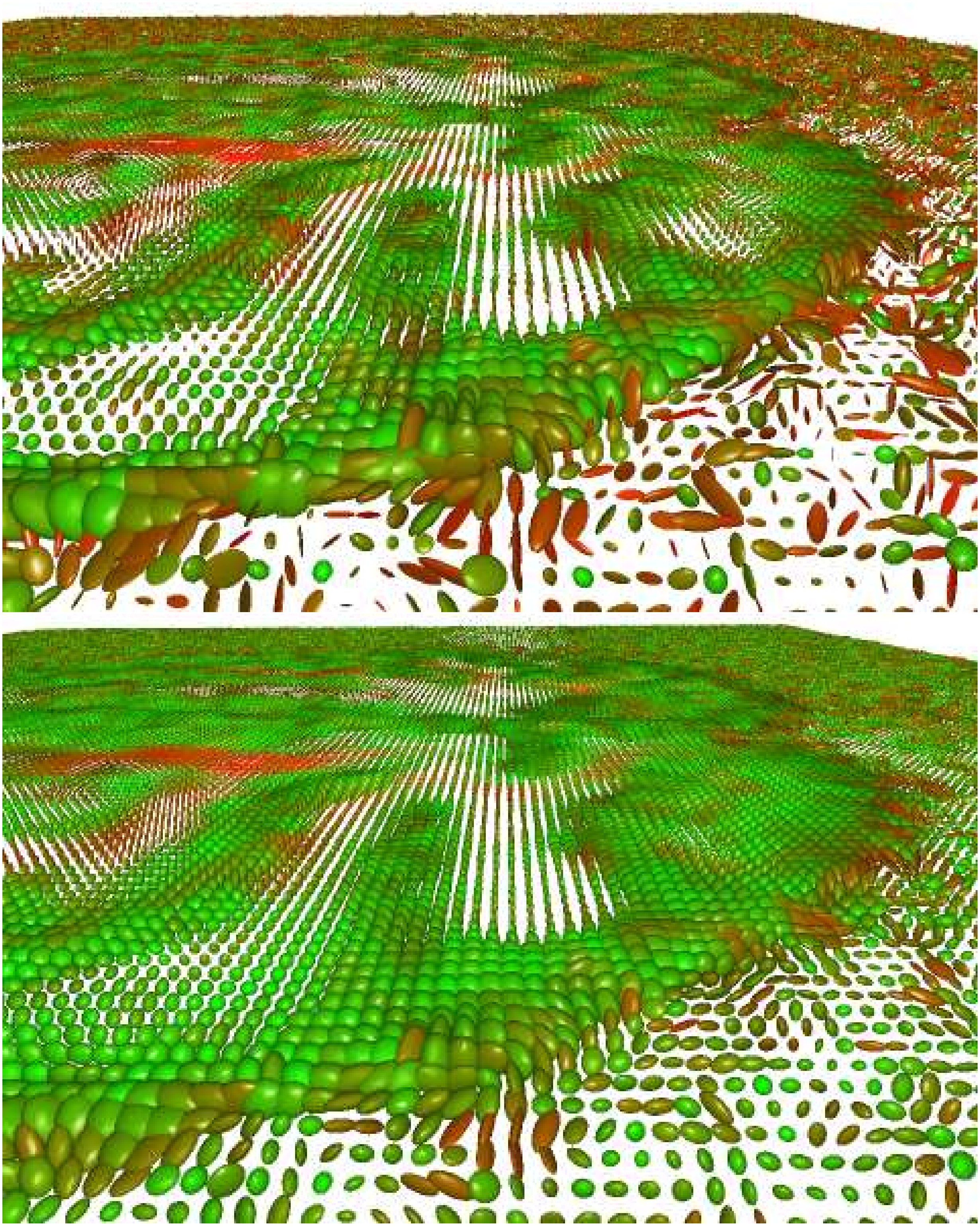

Figure 1: *Top:* One slice of artificially distorted tensor data $\vec{u} : \mathbb{R}^2 \to \mathbb{R}^9 \cong \mathbb{R}^{3\times3}$ of the human brain. *Bottom:* Filtered data.

Journal Articles

[P7-J01] T. Fidler, M. Grasmair, H. Pottmann, and O. Scherzer. Inverse Problems of Integral Invariants and Shape Signatures. submitted to *Inverse Problems*.

[P7-J02] F. Frühauf, H. Grossauer. Solving Constraint Ill-Posed Problems using Ginzburg-Landau Regularization Functionals. *J. Inverse Ill-Posed Probl.*, 2007. to appear.

[P7-J03] M. Haltmeier, A. Leitao, and O. Scherzer. Kaczmarz Methods for Regularizing Nonlinear Ill-Posed Equations I: Convergence Analysis. *Inverse Problems and Imaging*, 1(2):289–298, 2007.

[P7-J04] B. Hofmann, B. Kaltenbacher, C. Pöschl, and O. Scherzer. A Convergence Rates Result in Banach Spaces with Non-Smooth Operators. *Inverse Problems*, 23(3):987–1010, 2007.

[P7-J05] E. Resmerita and O. Scherzer. Error Estimates for Non-Quadratic Regularization and the Relation to Enhancing. *Inverse Problems*, 22:801–814, 2006.

[P7-J06] O. Scherzer, W. Yin, and S. Osher. Slope and G-set Characterization of Set-Valued Functions and Applications to Non-Differentiable Optimization Problems. *Comm. Math. Sci.*, 3:479–492, 2005.

[P7-J07] M. Haltmeier, R. Kowar, A. Leitao, and O. Scherzer. Kaczmarz Methods for Regularizing Non-linear Ill-Posed Equations II: Applications. *Inverse Problems and Imaging*, 1:507–523, 2007.

[P7-J08] T. Fidler, M. Grasmair, and O. Scherzer. Theory of Integral Invariants and Shape Signatures. submitted to *Inverse Problems*.

Refereed Articles in Books and Conference Proceedings

[P7-B01] C. Pöschl and O. Scherzer. Characterization of Minimizers of Convex Regularization Functionals. In *AMS-SIAM Special Session on Frames and Operator Theory in Analysis and Signal Processing*, Contemporary Mathematics, 2007. to appear.

Monographs

[P7-M01] O. Scherzer, M. Grasmair, H. Grossauer, F. Lenzen, and M. Haltmeier. Variational Methods in Imaging. Springer, 2007. submitted.

Invited Talks

[P7-I01] O. Scherzer. Non-Convex Regularization for Inverse Problems. Applied Inverse Problems, London (UK), 2005.

[P7-I02] O. Scherzer. Variational Principles for Denoising. PDE-Based Image Processing and Related Inverse Problems, Oslo (Norway), 2005.

[P7-I03] O. Scherzer. Thermoakustische Tomografie. Fakultätskolloquium der Fakultät für Mathematik der Technischen Universität München, München (Germany), 2005.

[P7-I04] O. Scherzer. Variationsmethoden zur Lösung von Bildverarbeitungsproblemen. Kolloquium: Institut für Numerische und Angewandte Mathematik (NAM), Göttingen (Germany), 2005.

[P7-I05] O. Scherzer. Variationelle Regularisierungsverfahren zur Lösung von Inversen Problemen und in der Bildverarbeitung, Freiburg (Germany), 2006.

[P7-I06] O. Scherzer. Thermoacoustic Tomography. Exploring the Frontiers of Dynamic Nuclear Medicine Imaging for Medical and Molecular Applications, Banff (Canada), 2006.

[P7-I07] O. Scherzer. Variational Methods for Image Processing and Inverse Problems. Seminar für Analysis und Numerik, Basel (Switzerland), 2006.

[P7-I08] O. Scherzer. Kaczmarz Methods for the Solution of Non-Linear Ill-Posed Equations. 1st International Congress of IPIA, Conference on Applied Inverse Problems 2007: Theoretical and Computational Aspects, Vancouver (Canada), 2007

[P7-I09] O. Scherzer. Some Aspects of Photoacoustic Imaging. 2nd MIP-LAMSIN Meeting on the Mathematics of Imaging (RIMA 07), Toulouse (France), 2007.

Contributed Talks and Poster Presentations

[P7-C01] M. Grasmair. Dual Settings for Total Variation Regularization. AMS National Meeting, New Orleans (USA), 2007.

[P7-C02] M. Grasmair. The Taut String Algorithm for Total Variation Regularization. Trends in Mathematical Imaging and Surface Processing, Oberwolfach (Germany), 2007.

[P7-C03] M. Grasmair. On an Inversion Problem for Integral Invariant Signatures. AIP 2007, Vancouver (Canada), 2007.

[P7-C04] M. Fuchs. NCBV: A Non-Convex Scale Space. Workshop "Inverse Problems", Obergurgl (Austria), 2005.

[P7-C05] M. Fuchs. A Non-Convex PDE Scale Space. Inverse Problems Reunion Conference, Lake Arrowhead (USA), 2005.

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[TD02] D. Tschumperlé and R. Deriche. Diffusion PDEs on Vector-Valued Images. *IEEE Signal Processing Magazine*, 19:16–25, 2002.

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