0.0.1 Report of project part 6

Title: Applications of Higher Geometries

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Summary

This project aims at the development of new methods in Industrial Geometry by applying techniques of classical geometry in combination with methods from approximation theory, numerical analysis and geometry processing. Those geometric concepts, which are known under the term higher geometries, include the various differential geometries (elementary Euclidean, affine and projective), and the representation of groups by their action on varieties of geometric objects, like Laguerre sphere geometry, line geometry, or kinematic spaces. An important aspect of the project is also the extension of these classical concepts to meet the needs in applications. Let us point here to the highlights of our work.

Motivated by applications in architectural design, we could define and study a new class of quadrilateral meshes – *conical meshes* – which discretize the network of principal curvature lines on a smooth surface. These meshes have planar faces and offset meshes at constant face/face distance, which turned out to have important applications for the beam layout in the actual construction of architectural freeform structures (Fig. 1). Conical meshes are an entity of Laguerre sphere geometry. The same holds for the new class of *edge-offset meshes*, whose definition comes from practical requirements on the supporting beam layout and whose study and computation exhibits relations to concepts such as circle patterns, Koebe polyhedra and Laguerre-isothermic surfaces. Shape limitations of quadrilateral edge-offset meshes motivated us to develop algorithms for computing approximate edge offsets and associated supporting beam layouts. Using ideas of relative differential geometry, we could introduce a wide class of practically interesting meshes with planar

faces (relative principal meshes) for which an adapted discrete curvature theory could be developed. As a result of these studies, we found *discrete differential geometry for architectural design* to be a wide area for future research. Some of the arising problems are currently investigated in two additional projects (funded by FWF and FFG/Waagner-Biro Stahlbau AG, respectively).

Another focus of research has been *geometric computing in shape spaces*. We presented a novel framework to treat shapes in the setting of Riemannian geometry. Shapes are considered as points in a shape space. We showed how to equip shape space with useful semi-Riemannian metrics and how to efficiently compute its geodesics. Working in shape space, various problems from geometric modeling and geometry processing can be treated in a consistent and unified way by linking them to geometric concepts such as parallel transport or the exponential map. These applications include shape morphing, deformation transfer and shape exploration.

As an extension of line geometry and Laguerre geometry, we studied the Euclidean *geometry of line elements*. This led to extensions of classical results and to algorithms for shape recognition and reconstruction. The geometry of surface elements and an associate *feature sensitive metric* on surfaces have been effectively used for feature sensitive geometry processing, such as mesh decimation or surface fitting, and for the detection, classification and editing of features.

Robust local shape descriptors related to curvatures on multiple scales (*integral invariants*) have been studied from a theoretical and computational perspective and could be successfully applied to *shape matching problems* such as registration or the automatic reassembly of fractured artefacts. Dynamic registration of rigid and articulated objects could be achieved in a space-time model exploiting kinematical properties. Shapes which can be partially matched onto themselves exhibit symmetries. Combined processing in a space of transformations between local neighborhoods and in the actual object space forms the basis of an algorithm for *symmetrization*, i.e., minimal deformation of a nearly symmetric shape toward a fully symmetric one.

Further topics addressed in this project include the approximation of a solid by a union of balls, applications in computational anatomy, Voronoi diagrams for oriented spheres, approximation in kinematic spaces, the computation of Minkowski sums and convolution surfaces, and studies of relations between Laguerre minimal surfaces and linear elasticity as a preparation for research tasks formulated in the present proposal.

Scientific Background / State of the art

Since the topics addressed in this project range across a large area of geometry, applied mathematics and geometric computing it is impossible to provide a good overview of the state of the art with respect to each individual topic. For important specific references on these topics, we refer to the discussion of the results provided below. Here, we just point to a few key references on areas treated in our project.

Kinematics and line geometry. The areas of kinematical geometry [BR90] and line geometry [PW01], which possess various relations to each other, are known as effective tools for robotics, the design and analysis of mechanisms and for statics. Recently, they could be effectively used in algorithms for surface recognition and reconstruction (see e.g. [PW01]). Based on a new relation between equiform kinematics and line element geometry, these classical relations could be extended [P6-J08] and the range of applications in surface recognition and reconstruction could be enlarged [P6-B04]. It is well known that the employment of appropriate point models for the group of rigid body motions (kinematic spaces) may simplify the solution of practical problems. This could be used in the work of the project (dynamic registration [P6-B06], penetration depth computation [P6-T03], discretization of the motion group [P6-T04]).

Shape space. Pioneered by Kendall [Ken84], shape spaces recently received increasing interest in the mathematical research community. We point to work by Michor and Mumford [MM06], which provides a theoretical background for our research. There are only a few contributions treating shape spaces and related topics from a *computational perspective.* Cheng at al. [CEF98] realized the intimate connection between shape spaces and deformations. A computational approach to spaces of curves was presented in [KSMJ04]. The gradient of a function on shape space depends on the metric. Hence the choice of metric is crucial for any shape evolution driven by a gradient descent algorithm of a geometric energy. This is the driving motivation behind recent efforts in the computer vision community [CFK05, YM05]. We also point to the forthcoming book on nonlinear geometry of nonrigid shapes [BBK07].

Sphere geometries. Differential geometry in the three classical sphere geometries of Möbius, Laguerre and Lie, respectively, is the subject of Blaschke's third volume on differential geometry [Bla29]; for a more modern treatment of some topics we refer to Cecil [Cec92]. Various applications of Laguerre geometry (rational offsets, rational parameterizations, surface reconstruction) have been presented by members of the group (M. Peternell, H. Pottmann). Within our project, sphere geometries, in particular the one of Laguerre, are present at many places, most notably in the work on discrete freeform structures for architecture [P6-J13, P6-B08, P6-J11, P6-J15], on Voronoi diagrams for oriented spheres [P6-B02], and on Laguerre minimal surfaces [P6-T06].

Discrete differential geometry. Most of the work relevant to our research concerns quadrilateral meshes with planar faces, which discretize so-called conjugate curve networks on surfaces [Sau70]. They are a basic concept in the integrable system viewpoint of discrete differential geometry [BS05]. Circular meshes, i.e., meshes whose quads possess a circum-circle, have been known as discrete counterpart to the network of principal curvature lines, formulated in terms of Möbius geometry. We added conical meshes [P6-J04, P6-J11, P6-J15] as Laguerre geometric counterparts. This inspired further work of Bobenko and Suris [BS07] and also led to a more general view on principal meshes formulated within relative differential geometry [P6-B08, P6-J13]. There is also a large body of research on discrete differential geometry of triangle meshes (see e.g. [DG06]) which we have used to estimate differential geometric quantities such as curvatures, but except for some results on curvatures of triangle meshes in isotropic geometry [P6-B08] we did not contribute to the theory.

Figure 1: Architectural design based on a conical mesh; part of this image appeared on the front cover of the SIGGRAPH 06 Proceedings issue of *ACM Transactions on Graphics*.

Results and Discussion

We discuss our work based on the activities formulated in the original proposal, although there are clearly strong relations and overlaps between these areas.

Geometry of shape manifolds.

Geometric computing in shape spaces opens up a new connection between geometry processing and Riemannian geometry [P6-J03, P6-T02]. Shapes – triangular meshes or more generally straight line graphs in Euclidean space – are treated as points in a shape space. We introduced useful semi-Riemannian metrics in this space to aid the user in design and modeling tasks, especially to explore the space of (approximately) isometric deformations of a given shape. Much of the work relies on an efficient algorithm to compute geodesics in shape spaces; to this end, we present a multi-resolution framework to solve the interpolation problem – which amounts to solving a boundary value problem – as well as the extrapolation problem – an initial value problem – in shape space. Based on these two operations, several classical concepts like parallel transport and the exponential map can be used in shape space to solve various geometric modeling and geometry processing tasks. Applications include shape morphing, shape deformation, deformation transfer, and intuitive shape exploration.

Approximation in kinematic spaces and shape spaces.

Kinematics has been present in a number of problem solutions such as in registration and shape matching [P6-J01]. Especially prominent is its appearance in a space-time hypersurface which has been effectively used for the dynamic registration of time-varying point clouds measured from rigid, articulated or slightly deforming objects [P6-B06].

Initiated by research of Manocha et al. [ZKVM06], we could present an algorithm for the efficient computation of generalized penetration depths [P6-T03]. Given a position A_0 of a 3D object A, penetrating object B, we want to minimally move A to resolve the penetration with B. Here we employ a distance in the motion group SE_3 which depends on a chosen reference object in the moving space [HPR04]. It is related to an embedding of SE_3 as a 6-dimensional manifold M^6 in a 12-dimensional Euclidean space associated with the affine transformations. The penetrating positions of A define a forbidden domain $F \subset M^6$, and thus penetration depth can be computed as the shortest distance (on M^6 or in R^{12}) from the given position $A_0 \in F$ to the boundary of F.

As a preparation for the computation of distance fields in the motion group, we have devised two ways for the discretization of SO(3) [P6-T04], one based on the Hopf fibration, the other using subdivision of the regular 600-cell in 4-space.

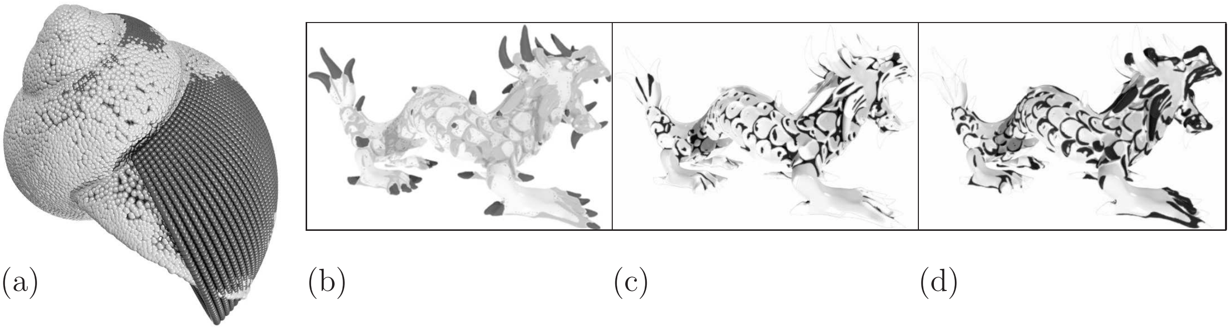
Related to kinematics, but also to shape deformations and Computational Geometry is the study of *symmetrization* by our postdoc researcher Niloy Mitra (jointly with M. Pauly, ETH Zürich and L. Guibas, Stanford) [P6-J07]. Their algorithm recognizes full or partial approximate symmetries and applies optimization strategies to obtain precise symmetry after minimal deformations. A key idea is the combined processing in a space of transformations between local neighborhoods and in the actual object space. This work is also very interesting for architectural design and will be merged with computational tools for discrete surfaces in architecture. It shall be continued in the larger context of recovering shape generating patterns in the second funding period.

Computational line and sphere geometry.

A line and an incident point is called a line element. In extension of line geometry, we studied the Euclidean geometry of line elements: Defining Plücker coordinates of line elements and linear complexes as line elements via a linear equation in these coordinates, we could show a remarkable relation to equiform kinematics: at any instant of a smooth one-parameter equiform motion the set of path normal elements of points lies in a linear line element complex [P6-J08]. This fact could be effectively used in 3D shape recognition and reconstruction [P6-B04]; in particular it allows one to empirically verify that the shapes of certain shells may be well approximated by spiral surfaces (surfaces generated by one-parameter subgroups of the group of similarities; cf. Fig. 2). These studies have been performed jointly with B. Odehnal, who then extended them in various ways, which eventually led to the submission of his habilitation in geometry.

A line element also determines an orthogonal surface element (normal plane plus the incident point) and thus line element geometry may also be seen as an extension of Laguerre geometry (see [P6-J08]). Another metric on surface elements, in particular those determined by the tangent planes of a smooth surface leads to the 'isophotic metric', whose first fundamental form is actually a combination of first and third fundamental forms of elementary differential geometry. The feature sensitive metric turned out to be a very valuable tool for feature sensitive geometry processing, such as mesh decimation or surface fitting, and for the detection, classification and editing of features (see [P6-J05, P6-J06] and Fig. 2). Related morphological methods have been used in M. Hofer's cooperation with the research group of G. Sapiro at the University of Minnesota that deals with the extraction of sulcal fundi in brain data [P6-J02, P6-B05].

The cooperation with Subproject 05 on Computational Geometry led to an algorithm



(a) (b) (c) (d)

Figure 2: (a) Reconstruction of axis and exact spiral surface approximating the shell of a snail using line element geometry. (b)–(c): The feature sensitive metric is a tool for feature sensitive geometry processing and feature classification. The series of images shows prongs (b), valleys (c), and ridges (d).

for the approximation of a solid by a union of balls [P6-B01]. We are currently investigating its application for Minkowski sum computations. Minkowski sums and convolutions are also the topic of papers by M. Peternell, partially written jointly with B. Jüttler [P6-J14, P6-J10, P6-J09]. A somehow related topic has been the measurement of cortical thickness, where we tried to fit balls into the respective layer (based on statistical methods) and use their radii as a value for local layer thickness [P6-T07]. Approximation by balls and the related extraction of the (stable part of the) medial axis will be further pursued in the second funding period due to important applications toward 3D meshing for CFD simulations.

In an attempt to extend fundamental concepts of Computational Geometry to higher geometries, we studied Voronoi diagrams of oriented spheres [P6-B02]. They turned out to be quite complicated for those distance measures which are related to the interpretation of oriented spheres as events (Laguerre geometry as a geometric model of special relativity). Further work is necessary here and planned for the second funding period of the NRN.

Differential sphere geometry has so far hardly been used for applications in geometric modeling or geometry processing. In an attempt to better exploit the power of the available theory [Bla29], we addressed the computation of Laguerre-minimal surfaces, i.e., the minimizers of the energy $\int (H^2 - K)/K dA$. We showed that these surfaces appear in the isotropic model of Laguerre geometry as graphs of biharmonic functions and thus provided a computational approach as well [P6-T06]. The actual goal of the research are constructions of discrete counterparts.

Computational differential geometry.

This activity may be seen as a replacement of activity 6.4 (differential morphology and kinematics); the latter has not been pursued due to the cancelation of personnel and of Subproject 04, and the resulting missing cooperation with M. Husty.

The extraction of curvature information for surfaces is a basic problem of Geometry Processing. An integral invariant solution of this problem can be based on integrals over local neighborhoods defined by kernel balls of various sizes. This approach is not only robust to noise, but also adjusts to the level of detail required. We have introduced and studied a number of such invariants with respect to the relation to classical invariants (via an asymptotic analysis), proved robustness results, proposed efficient implementations and introduced applications to various shape matching problems [P6-J12, P6-T05]. A highlight has been the SIGGRAPH 2006 paper on reassembling fractured objects [P6-J01].

Given a shape, e.g. a curve, one may compute at least two different integral invariants (I_1, I_2) for each of its points and thereby define another planar curve, a so-called shape signature. This follows an approach by Manay et al. [MHYS04]. The signature is not effected under those transformations which do not change the invariants, which is a great advantage for shape retrieval or shape matching. However, almost nothing is known whether it is possible to (at least theoretically) reconstruct a shape from a certain signature. A few initial results have been achieved by the group of O. Scherzer; more research in this largely unexplored area is planned for the second funding period.

An unexpectedly high success has been our paper on geometric modeling with conical meshes and developable surfaces [P6-J04], which appeared at SIGGRAPH 2006. Not only did we receive outstanding reviews and enthusiastic feedback from colleagues in the graphics community, the work was also stimulating for mathematicians specializing in Discrete Differential Geometry (e.g. A. Bobenko, TU Berlin, Yu. Suris, TU Munich). It opened a new direction of research which solves practical problems, e.g. from architecture (for a survey see [P6-B07]), by combining and extending methods from discrete differential geometry, classical geometry and geometric computing (e.g. geometric optimization). On the theoretical side, we obtained new results on principal meshes, i.e., discrete counterparts to the network of principal curvature lines, and on discrete minimal surfaces; the results have been accepted for publication in mathematical journals [P6-J11, P6-J15]. On the practical side, we solved for the first time the difficult problem of designing freeform structures for architecture with planar quadrilateral faces and torsion free nodes in the supporting beam layout. Moreover, the proposed meshes possess precise discrete offsets which allow for the construction of multi-layer structures. Our latest results could be published at SIGGRAPH 2007 [P6-J13] and in an invited paper for the SMAI-AFA conference 'Curves and Surfaces' [P6-B08].

International architects and construction companies showed great interest in this innovation. We received funding for a project (FIT-IT Project 813391) with Waagner-Biro Stahlbau AG, Vienna, which won the second prize among all proposals submitted to FFG in the category 'Visual Computing'. Moreover, we entered a funded cooperation with RFR, Paris, on architectural applications of special semi-discrete surface models. The theory of such models shall be investigated in the second funding period of the NRN (Subproject 09).

National and international cooperation

We are very glad that the work within this project strongly improved our international cooperations:

Initiated by our work on conical meshes, we started a very fruitful cooperation with Alexander Bobenko (TU Berlin) [P6-J13]; we are currently in the stage of writing a joint publication on discrete curvatures for quad meshes with planar faces.

We could continue our cooperation with Leonidas Guibas and his coworkers at Stanford University. Several joint papers (within the present project: [P6-J01, P6-J07, P6-B06] and the employment of Niloy Mitra, who received his PhD at Stanford, as postdoc researcher in the NRN result from this cooperation. Niloy Mitra has a cooperation with Mark Pauly (ETH Zürich) (see [P6-J07]) which shall be extended to a cooperation between the two research groups (see proposal for second funding period).

We benefitted a lot from cooperations with the research groups of Wenping Wang, University of Hong Kong (results: [P6-J04, P6-J13, P6-B08]) and Shi-Min Hu, Tsinghua University, Beijing (results: [P6-J01, P6-J04, P6-J05, P6-J06, P6-J12, P6-T05]).

Furthermore, the traditional exchange of ideas with Bahram Ravani, University of California at Davis, stimulated our work on penetration depth computation [P6-T03]. Our earlier contributions on energy minimizing splines in manifolds initiated a cooperation with Guillermo Sapiro, University of Minnesota, where Michael Hofer has been working as a postdoctoral researcher (results: [P6-J02, P6-B05]).

On a national level, part of the research in the present project has been linked to work within the Kplus competence center *Acvanced Computer Vision*, Vienna. The cooperation focussed on medical applications and 3D registration.

As already mentioned, we receive important input for our research in architectural geometry from cooperations with Waagner-Biro Stahlbau AG, Vienna and RFR, Paris.

Cooperations within Austria are all within the present NRN and described at many places of the present document. There have been joint publications with all research groups represented in the NRN ([P6-J14, P6-B01, P6-B02, P6-T01], not mentioning the large number of joint papers of J. Wallner, TU Graz, with the group at TU Wien).

Journal articles

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[P6-I27] H. Pottmann. Architectural Geometry. SIAM Conf. Geometric Design and Computing, San Antonio, Texas, 4.-8.11.2007 (invited plenary lecture)

[P6-I28] J. Wallner. What can Geometry do for you? Akademie der bildenden Künste, Wien. 30.11.2005.

[P6-I29] J. Wallner. Computing quadrilateral and conical meshes, Workshop "Discrete Differential Geometry", Oberwolfach, 06.03.-10.03.2006

[P6-I30] J. Wallner. Diskrete Differentialgeometrie. TU Wien, Vortragsreihe: Wissenswertes aus der Mathematik, 16.10.2006.

[P6-I31] J. Wallner. Diskrete Geometrie und Freiformflächen in der Architektur, FoSP-Kolloquium, TU Graz, 16.03.2007.

[P6-I32] J. Wallner. Geometry of Multilayer Freeform Structures for Architecture, Discrete Differential Geometry conference, Berlin, 16.–20.07.2007.

Contributed talks and poster presentations

[P6-C00] P. Grohs. Laguerre minimal surfaces and isotropic geometry. Workshop 'Polyhedral Surfaces and Industrial Applications', Strobl, 15.-18.9.2007

[P6-C01] M. Hofer. A geometric method for automatic extraction of sulcal fundi, FSP S92 Industrial Geometry Meeting, Graz (Austria), 23.3.2007.

[P6-C02] M. Hofer. Reassembling broken objects, FSP S92 Industrial Geometry Meeting, Strobl (Austria) 20.6.2006.

[P6-C03] M. Hofer. Reassembling fractured objects by geometric matching, Geometry Seminar, University of Minnesota, Minneapolis, 12.4.2006.

[P6-C04] M. Hofer. Solving 3D Puzzles, Advanced Computer Vision Colloquium, Vienna (Austria), 31.3.2006.

[P6-C05] M. Hofer. A geometric method for automatic extraction of sulcal fundi, Advanced Computer Vision Colloquium, Vienna (Austria), 216.12.2005.

[P6-C06] M. Hofer. Line Element Geometry for 3D Shape Understanding and Reconstruction, FSP S92 Industrial Geometry Meeting, St. Martin (Austria), 23.11.2005.

[P6-C07] M. Hofer. 3D Shape Recognition and Reconstruction Based on Line Element Geometry, 9th SIAM Conf. Geometric Design and Computing, Phoenix, Arizona, 2.11.2005.

[P6-C08] M. Hofer. 3D Shape Recognition and Reconstruction with Line Element Geometry. IMA Workshop 'New Mathematics and Algorithms for 3-D Image Analysis', Minneapolis, 9.1.2006 (Poster).

[P6-C09] M. Kilian. 3D Shape Morphing Based On Geodesics In Shape Spaces. 1st FSP-Meeting (St. Martin) 23-25 November, 2005

[P6-C10] M. Kilian. 3D shape warping based on Geodesics in shape space. Shape Space Workshop at IMA (Poster). April 3-7 2006

[P6-C11] M. Kilian. Quasi-isometric shape interpolation. 2nd FSP-Meeting (Strobl) 19-22 June, 2006

[P6-C12] M. Kilian. Geometric Modeling in Shape Space. Paper presentation at SIGGRAPH'07, San Diego, Calif., 8.8.2007.

[P6-C13] N. J. Mitra. Symmetrization, FSP Industrial Geometry workshop, Graz, 22. 3. 2007

[P6-C14] N. J. Mitra. Dynamic Geometry Registration, Symposium on Geometry Processing, Barcelona, 2007.

[P6-C15] N. J. Mitra. Symmetrization. Paper presentation at SIGGRAPH 2007, San Diego, Calif., 08/2007.

[P6-C16] N. J. Mitra. Symmetry Detection and Symmetrization. Workshop 'Polyhedral Surfaces and Industrial Applications', Strobl, 15.-18.9.2007

[P6-C17] G. Nawratil. Distance computations in kinematics. FSP Meeting, Obergurgl (Austria), 2006.

[P6-C18] M. Peternell. A geometric idea to solve the eikonal equation, Spring conf. computer graphics 2005, Budmerice, Slovakia, 12. 05. 2005.

[P6-C19] M. Peternell. Constrained Surface Approximation from a Geometric Optimization Perspective, SIAM Conf. Geom. Design and Computing, Mesa (AZ), 1.11. 2005.

[P6-C20] M. Peternell. A geometric idea to solve the eikonal equation, Spring conf. on Computer Graphics 2005, Budmerice, Slovakia, 12. 05. 2005.

[P6-C21] M. Peternell. Envelopes of Moving Solids, 2005 SIAM Conf. Geometric Design and Computing, Mesa, Arizona, 02.11. 2005.

[P6-C22] M. Peternell. Boundary Surfaces of Moving Objects, 'Industry Challenges in Geometric Modeling and CAD – 2006', Darmstadt, 09.–10.03.2006.

[P6-C23] M. Peternell. Boundary Surfaces of Moving Objects, 'Curves and Surfaces – 2006', Avignon, 29.6.–05.07.2006.

[P6-C24] M. Peternell. Rational Offset Surfaces and Related Problems, 'Automated Deduction in Geometry', Pontevedra, Spain, 31.08.–02.09. 2006.

[P6-C25] M. Peternell. Sphere-geometric aspects of bisector surfaces, 'Algebraic Geometry and Geometric Modeling', Barcelona, Spain, 04.09.–07.09. 2006.

[P6-C26] H. Pottmann. Laguerre minimal surfaces and thin plate splines. Seminar on Industrial Geometry, Obergurgl, Austria, 5.9.2006.

[P6-C27] J. Wallner. Fair Curve Networks and their applications. Österreichische Mathematikertagung 2005, Klagenfurt, 18.–23.09.2005.

[P6-C28] J. Wallner. Geometric Modeling with Conical Meshes and Developable Surfaces Statuseminar des FSP “Industrielle Geometrie”, Strobl, 19.-22.06.2006.

[P6-C29] J. Wallner. Robust Principal Curvatures on Multiple Scales. Conference “4th European Symposium on Geometry Processing”, Cagliari, 26.–28.06.2006.

[P6-C30] J. Wallner. Geometric Modeling with Conical Meshes and Developable Surfaces, paper presentation at SIGGRAPH 2006, Boston, 30.07.–03.08.2006.

[P6-C31] J. Wallner. New results on circular and conical meshes. 2. Statusseminar des FSP “Industrielle Geometrie”, Obergurgl, 04.–05.09.2006

[P6-C32] J. Wallner. Meshes, offset properties, and Higher Geometries. Workshop des Nationalen Forschungssnetzwerks “Industrielle Geometrie”, Vorau, 27.–29.11.2006

[P6-C33] J. Wallner. Discrete Surfaces with applications in architectural design. ACV-Kolloqium, Wien, 12.12.2006.

[P6-C34] J. Wallner. Discrete Differential Geometry, 4. Statusseminar des FSP “Industrielle Geometrie”, St. Martin, 22.-23.03.2007.

[P6-C35] J. Wallner. Geometry of Multilayer Freeform Structures for Architecture. paper presentation at SIGGRAPH 2007, 5.–9.8.2007

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